Kinematika

Anyagi pont kinematikája

mozgástörvény: $\tilde{\mathbf{r}}\,(s(t)) = \mathbf{r}(t) = \mathbf{r}(t_0) + \int_{t_0}^{t} \mathbf{v}(\tau)\,\mathrm{d}\tau,$ $\quad \mathbf{v}(t) = \dot{\mathbf{r}}(t) = \dot{s}(t)\mathbf{e}_t = v\mathbf{e}_t$

$\mathbf{v}(t) = \mathbf{v}(t_0) + \int_{t_0}^{t} \mathbf{a}(\tau)\,\mathrm{d}\tau,$ $\quad \mathbf{a}(t) = \dot{\mathbf{v}}(t) = \ddot{\mathbf{r}}(t) = \ddot{s}\mathbf{e}_t + v^2\frac{1}{\rho}\mathbf{e}_n = a_t\mathbf{e}_t + a_n\mathbf{e}_n$

Merev test kinematikája

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{AB},$ $\quad \mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\varepsilon} \times \mathbf{r}_{AB} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{AB}),$ $\quad \boldsymbol{\varepsilon} = \dot{\boldsymbol{\omega}}$

Síkmozgás esetén:

P: sebességpólus ($\mathbf{v}_P = \mathbf{0}$)

$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_{PA}$

$\mathbf{r}_{AP} = \frac{1}{\omega^2}(\boldsymbol{\omega} \times \mathbf{v}_A)$

$\overline{AP} = \frac{v_A}{\omega}$

$\mathbf{u} = \frac{1}{\omega^2}(\boldsymbol{\omega} \times \mathbf{a}_P)$

G: gyorsuláspólus ($\mathbf{a}_G = \mathbf{0}$)

$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\varepsilon} \times \mathbf{r}_{AB} - \omega^2\mathbf{r}_{AB}$

$\mathbf{a}_G = \mathbf{0} = \mathbf{a}_A + \boldsymbol{\varepsilon} \times \mathbf{r}_{AG} - \omega^2\mathbf{r}_{AG}$

$\mathbf{r}_{AG} = \frac{1}{\varepsilon^2 + \omega^4}(\omega^2\mathbf{a}_A + \boldsymbol{\varepsilon} \times \mathbf{a}_A)$

$\mathrm{tg}\,\alpha = \frac{\varepsilon}{\omega^2}$

Mozgás leírása egymáshoz képest mozgó koordinátarendszerekben

$\mathbf{v}_P = \boldsymbol{\beta}_P + \mathbf{v}_{P\text{száll}}$ $\quad \boldsymbol{\omega}_{20} = \boldsymbol{\omega}_{21} + \boldsymbol{\omega}_{10}$ $\quad (\boldsymbol{\omega}_{\text{absz}} = \boldsymbol{\omega}_{\text{rel}} + \boldsymbol{\omega}_{\text{száll}})$

$\mathbf{a}_P = \boldsymbol{\alpha}_P + \mathbf{a}_{P\text{száll}} + \mathbf{a}_{P\text{Cor}}$ $\quad \boldsymbol{\varepsilon}_{20} = \boldsymbol{\varepsilon}_{21} + \boldsymbol{\varepsilon}_{10} + \boldsymbol{\omega}_{10} \times \boldsymbol{\omega}_{21}$ $\quad (\boldsymbol{\varepsilon}_{\text{absz}} = \boldsymbol{\varepsilon}_{\text{rel}} + \boldsymbol{\varepsilon}_{\text{száll}} + \boldsymbol{\omega}_{\text{száll}} \times \boldsymbol{\omega}_{\text{rel}})$

$\mathbf{a}_{P\text{Cor}} = 2\boldsymbol{\omega}_{10} \times \boldsymbol{\beta}_P$

Dinamika

Anyagi pont dinamikája

anyagi pont
- *impulzusa* (mozgásmennyisége vagy lendülete): $\mathbf{I} = m\mathbf{v}$
- A pontra vonatkozó *perdülete* (impulzus nyomatéka): $\boldsymbol{\Pi}_A = \mathbf{r}_{AP} \times \mathbf{I}$
- *kinetikus* (mozgási) *energiája*: $\mathcal{T} = \frac{1}{2}mv^2$

az anyagi pontra ható $\mathbf{F}$ erő
- teljesítménye: $\mathcal{P} = \mathbf{F} \cdot \mathbf{v}$
- munkája a $[t_0, t_1]$ időintervallumon: $\mathcal{W}_{01} = \int_{t_0}^{t_1} \mathcal{P}\,\mathrm{d}t$
- potenciálos erők esetén: $\mathbf{F} = -\,\mathrm{grad}\,\mathcal{U}$

  $\mathcal{W}_{01} = -\mathcal{U}_1 + \mathcal{U}_0 = \mathcal{U}_0 - \mathcal{U}_1$

Tételek:

impulzus-tétel: $\dot{\mathbf{I}} = \mathbf{F}\ \left(= \sum_{i=1}^{n} \mathbf{F}_i\right)$ $\quad \left(\int_{t_0}^{t_1} \dot{\mathbf{I}}\,\mathrm{d}t =\right) \mathbf{I}_1 - \mathbf{I}_0 = \int_{t_0}^{t_1} \mathbf{F}\,\mathrm{d}t$

perdület-tétel: $\mathbf{D}_A = \mathbf{M}_A\ \left(= \sum_{i=1}^{n} \mathbf{M}_{Ai}\right)$ $\quad \boldsymbol{\Pi}_{A1} - \boldsymbol{\Pi}_{A0} = \int_{t_0}^{t_1} \mathbf{M}_A\,\mathrm{d}t\,,$ ha $\mathbf{v}_A = \mathbf{0}$

($\mathbf{D}_A = \dot{\boldsymbol{\Pi}}_A + \mathbf{v}_A \times \mathbf{I}$ a kinetikai nyomaték vektor)

egybefoglalva: $\left[\dot{\mathbf{I}}; \mathbf{D}_A\right]_A = [\mathbf{F}; \mathbf{M}_A]_A$

teljesítménytétel: $\dot{\mathcal{T}} = \mathcal{P}$

munkatétel: $\mathcal{T}_1 - \mathcal{T}_0 = \left(\int_{t_0}^{t_1} \dot{\mathcal{T}}\,\mathrm{d}t = \int_{t_0}^{t_1} \mathcal{P}\,\mathrm{d}t =\right) \mathcal{W}_{01}$

potenciálos erők esetén: $\mathcal{T}_1 - \mathcal{T}_0 = \mathcal{U}_0 - \mathcal{U}_1\ (= \mathcal{W}_{01})\,,$ azaz $\mathcal{T}_1 + \mathcal{U}_1 = \mathcal{T}_0 + \mathcal{U}_0$

Anyagi pont „relatív” dinamikája: $m\boldsymbol{\alpha} = \mathbf{F}_{\text{rel}}\ (= \mathbf{F} + \mathbf{F}_{\text{száll}} + \mathbf{F}_{\text{Cor}})$

$\mathbf{F}_{\text{száll}} = -m\mathbf{a}_{\text{száll}}\,,$ $\quad \mathbf{F}_{\text{Cor}} = -m\mathbf{a}_{\text{Cor}}$

Anyagi pontrendszer dinamikája

$\mathbf{F}_i$: az i-edik anyagi pontra ható *külső* erők eredője

$\mathbf{B}_{ij}$: a j-edik anyagi pontról az i-edikre ható *belső* erő

$\mathbf{M}_A$: *külső* erők nyomatéka az A pontra

Impulzus: $\mathbf{I} = m\mathbf{v}_S$, ahol $m = \sum_{i=1}^{n} m_i$, $\mathbf{v}_S = \frac{1}{m}\sum_{i=1}^{n} m_i\mathbf{v}_i$

Perdület: $\boldsymbol{\Pi}_A = \sum_{i=1}^{n} \mathbf{r}_{Ai} \times (m_i\mathbf{v}_i)$

Kinetikus energia: $\mathcal{T} = \sum_{i=1}^{n} \frac{1}{2} m_i v_i^2$

Tételek: $\left[\dot{\mathbf{I}}; \mathbf{D}_A\right]_A = [\mathbf{F}; \mathbf{M}_A]_A$, ahol $\mathbf{D}_A = \dot{\boldsymbol{\Pi}}_A + \mathbf{v}_A \times \mathbf{I}$ $\quad$ ($\mathbf{D}_A = \dot{\boldsymbol{\Pi}}_A$, ha $\mathbf{v}_A = \mathbf{0}$ vagy $A \equiv S$)

$\dot{\mathcal{T}} = \mathcal{P}_F + \mathcal{P}_B$

$\mathcal{T}_1 - \mathcal{T}_0 = \mathcal{W}_{01}$

Merev test dinamikája

Impulzus: $\mathbf{I} = m\mathbf{v}_S$

Perdület - általánosan: $\boldsymbol{\Pi}_A = \boldsymbol{\Theta}_A \cdot \boldsymbol{\omega} + \mathbf{r}_{AS} \times m\mathbf{v}_A$ $\quad$ ($A = S: \ \boldsymbol{\Pi}_S = \boldsymbol{\Theta}_S \cdot \boldsymbol{\omega}$)

- $S \to A$: $\boldsymbol{\Pi}_A = \boldsymbol{\Pi}_S + \mathbf{r}_{AS} \times \mathbf{I} = \boldsymbol{\Theta}_S \cdot \boldsymbol{\omega} + \mathbf{r}_{AS} \times m\mathbf{v}_S$

Tehetetlenségi nyomaték: $\boldsymbol{\Theta}_A = \boldsymbol{\Theta}_S + \boldsymbol{\Theta}_{AS}$, ahol

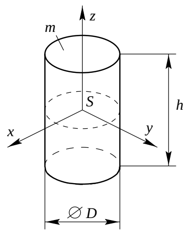
$$\boldsymbol{\Theta}_S = \begin{bmatrix} \Theta_x & -D_{xy} & -D_{xz} \\ -D_{yx} & \Theta_y & -D_{yz} \\ -D_{zx} & -D_{zy} & \Theta_z \end{bmatrix}_{(x,y,z)} = \begin{bmatrix} \Theta_1 & 0 & 0 \\ 0 & \Theta_2 & 0 \\ 0 & 0 & \Theta_3 \end{bmatrix}_{(1,2,3)}, \ \boldsymbol{\Theta}_{AS} = m \begin{bmatrix} y_{AS}^2 + z_{AS}^2 & -x_{AS}\,y_{AS} & -x_{AS}\,z_{AS} \\ -y_{AS}\,x_{AS} & x_{AS}^2 + z_{AS}^2 & -y_{AS}\,z_{AS} \\ -z_{AS}\,x_{AS} & -z_{AS}\,y_{AS} & x_{AS}^2 + y_{AS}^2 \end{bmatrix}$$

továbbá: $\Theta_x = \int_{(m)} \left(y^2 + z^2\right) \mathrm{d}m$ - az x tengelyre számított tehetetlenségi nyomaték,

$D_{xy} = D_{yx} = \int_{(m)} xy\,\mathrm{d}m$ - a deviációs tehetetlenségi nyomaték.

Henger:

$R = \dfrac{D}{2}$



$\Theta_x = \Theta_y = \frac{1}{4}mR^2 + \frac{1}{12}mh^2$

$\Theta_z = \frac{1}{2}mR^2$

korong: ($h \ll R$) $\quad \Theta_x = \Theta_y = \frac{1}{4}mR^2, \quad \Theta_z = \frac{1}{2}mR^2$

prizmatikus rúd: ($R \ll h = l$) $\quad \Theta_x = \Theta_y = \frac{1}{12}ml^2, \quad \Theta_z = 0$

Kinetikus energia: $\boxed{\mathcal{T} = \frac{1}{2}mv_S^2 + \frac{1}{2}\boldsymbol{\omega}^\top \cdot \boldsymbol{\Theta}_S \cdot \boldsymbol{\omega}} = \frac{1}{2}\left(\mathbf{I} \cdot \mathbf{v}_S + \boldsymbol{\omega} \cdot \boldsymbol{\Pi}_S\right)$

Ha $\mathbf{v}_A = \mathbf{0}$: $\mathcal{T} = \frac{1}{2}\boldsymbol{\omega}^\top \cdot \boldsymbol{\Theta}_A \cdot \boldsymbol{\omega}$,

ha $\boldsymbol{\Pi}_S \| \boldsymbol{\omega}$: $\mathcal{T} = \frac{1}{2}\left(mv_S^2 + \Theta_S\omega^2\right)$

„Teljesítmény": $\mathcal{P} = \sum_{i=1}^{n} \mathbf{F}_i \cdot \mathbf{v}_i + \mathbf{M} \cdot \boldsymbol{\omega}$

Tételek: $\left[\dot{\mathbf{I}}; \mathbf{D}_A\right]_A = [\mathbf{F}; \mathbf{M}_A]_A$

$\dot{\mathcal{T}} = \mathcal{P}$

$\mathcal{T}_1 - \mathcal{T}_0 = \mathcal{W}_{01}$

$\mathbf{D}_S = \boldsymbol{\Theta}_S \cdot \boldsymbol{\varepsilon} + \boldsymbol{\omega} \times \boldsymbol{\Pi}_S = \boldsymbol{\Theta}_S \cdot \boldsymbol{\varepsilon} + \boldsymbol{\omega} \times (\boldsymbol{\Theta}_S \cdot \boldsymbol{\omega})$

$\mathbf{D}_A = \boldsymbol{\Theta}_A \cdot \boldsymbol{\varepsilon} + \boldsymbol{\omega} \times (\boldsymbol{\Theta}_A \cdot \boldsymbol{\omega}) + \mathbf{r}_{AS} \times m\mathbf{a}_A$

Ütközések

Centrikus ütközés ($\mathbf{r}_{S_1S_2} = \lambda\mathbf{n}$ az ütközési normálisba esik):

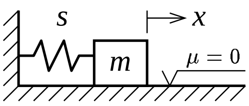
impulzus tétel: $m_1v_1 + m_2v_2 = m_1c_1 + m_2c_2 = (m_1 + m_2)c_S$ $\quad (v_S = c_S)$

ütközési tényező: $-k = \dfrac{v_1 - v_S}{c_1 - c_S} = \dfrac{v_2 - v_S}{c_2 - c_S} \Longrightarrow \boxed{v_i = c_S + k(c_S - c_i)}$

	Álló tengely (A) körül forgó test ütközése	Excentrikus ütközés
talppont (T) helye az ütközés normálisán:	$\mathbf{r}_{AT} \cdot \mathbf{n} = 0$	$\mathbf{r}_{ST} \cdot \mathbf{n} = 0$
lökésközéppont (K) helye :	$(\ldots)$	$\mathbf{r}_{SK} \cdot \mathbf{r}_{ST} = -\frac{1}{m}\Theta_S$ $\quad (\mathbf{c}_K = \mathbf{v}_K = \mathbf{v}_S + \boldsymbol{\omega} \times \mathbf{r}_{SK})$
redukció:	$m_T = \dfrac{\Theta_A}{\mathbf{r}_{AT}^2}$	$m_T = m\dfrac{\Theta_S}{\Theta_S + m\mathbf{r}_{ST}^2}$
	$\mathbf{c}_T = \boldsymbol{\Omega} \times \mathbf{r}_{AT}$	$\mathbf{c}_T = \mathbf{c}_S + \boldsymbol{\Omega} \times \mathbf{r}_{ST}$
	$\mathbf{v}_T = \boldsymbol{\omega} \times \mathbf{r}_{AT}$	$\mathbf{v}_T = \mathbf{v}_S + \boldsymbol{\omega} \times \mathbf{r}_{ST}$

Egyszabadságfokú lengőrendszerek

Csillapítatlan szabad lengések



(x=0 helyen a rugó erőmentes)

$m\ddot{x} + sx = 0$

DE: $\boxed{\ddot{x} + \alpha^2 x = 0}$

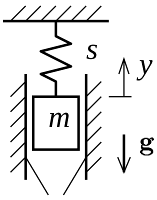
KF: $x(0) = x_0 \quad \dot{x}(0) = v_0$

lengésidő, frekvencia: $T = 2\pi/\alpha\,, \qquad \nu = 1/T$

$x_h(t) = C_1 \cos\alpha t + C_2 \sin\alpha t = B\sin(\alpha t + \delta)$

$C_1 = x_0 \quad C_2 = v_0/\alpha \qquad B = \sqrt{C_1^2 + C_2^2} \qquad \operatorname{tg}\delta = C_1/C_2$

nehézségi erőtérben (y=0 helyen a rugó erőmentes):



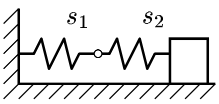
$\mu = 0$

$m\ddot{y} + sy = -mg \qquad \boxed{\ddot{y} + \alpha^2 y = -g} \qquad y_{ih}(t) = y_h(t) + y_p(t) = B\sin(\alpha t + \delta) - \frac{mg}{s}$

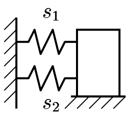
statikus egyensúlyi helyzet: $y_{st} = -mg/s$

$x = y - y_{st} = y + \frac{mg}{s}, \quad \ddot{x} = \ddot{y}, \quad \ddot{x} + \alpha^2 x = 0 \qquad x_h(t) = B\sin(\alpha t + \delta)$

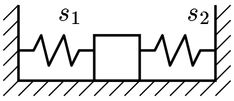
rugókapcsolódások:



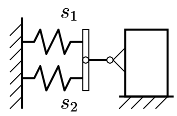
$\frac{1}{s} = \frac{1}{s_1} + \frac{1}{s_2}$



vagy

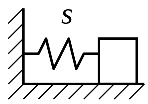


$s = s_1 + s_2$



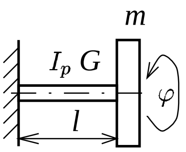
$\frac{4}{s} = \frac{1}{s_1} + \frac{1}{s_2}$

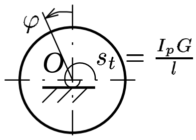
$\Rightarrow$



$\Rightarrow s$

torziós lengések:





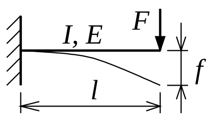
$= \frac{I_p G}{l}$

$\Theta_O\ddot{\varphi} + s_t\varphi = 0$

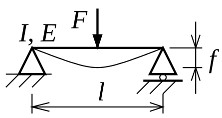
$\ddot{\varphi} + \alpha^2\varphi = 0$

$\varphi(t) = B\sin(\alpha t + \delta)$

hajlító lengéshez járulékképletek:



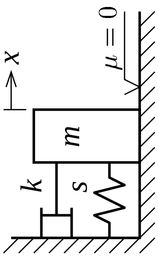
$f = \frac{Fl^3}{3IE}$



$f = \frac{Fl^3}{48IE}$

Csillapított szabad lengések

sebességgel arányos csillapítás (x=0 helyen a rugó erőmentes):



$m\ddot{x} + k\dot{x} + sx = 0$

$\boxed{\ddot{x} + 2D\alpha\dot{x} + \alpha^2 x = 0}$

$x(t) = \Re(Ae^{\lambda t})$ (valós r.)

kar. egyenlet:

$\lambda^2 + 2D\alpha\lambda + \alpha^2 = 0$

$D < 1: \quad x(t) = e^{-D\alpha t}(C_1\cos\gamma t + C_2\sin\gamma t) \equiv Be^{-D\alpha t}\sin(\gamma t + \delta)$ „lengő”

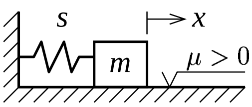
$\gamma = \alpha\sqrt{1 - D^2}, \quad T = \frac{2\pi}{\gamma}, \quad \Lambda = \ln\frac{A_n}{A_{n+2}} = \ln\frac{x(t)}{x(t+T)} = D\alpha T$

$D = 1: \quad x(t) = e^{-\alpha t}(C_1 + C_2 t)$ „kritikus” csill.-ú

$D > 1: \quad x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \equiv e^{-D\alpha t}(\tilde{C}_1 \operatorname{ch}|\gamma|t + \tilde{C}_2 \operatorname{sh}|\gamma|t)$ „túlcsill.”

$\lambda_{1,2} = -D\alpha \pm \alpha\sqrt{D^2 - 1} \qquad \lambda_1 < -\alpha < \lambda_2 < 0$

Coulomb-súrlódással csillapított lengések:



$m\ddot{x} = -sx - \mu mg \operatorname{sgn}\dot{x} \qquad \boxed{\ddot{x} + \alpha^2 x = -f_0\alpha^2 \operatorname{sgn}\dot{x}} \qquad T = \frac{2\pi}{\alpha}, \quad f_0 = \frac{\mu mg}{s}$

Pl.: $x(0) = x_0 > 0, \quad \dot{x}(0) = 0$

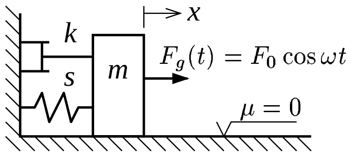
$\leftarrow \quad \dot{x} < 0 \quad (0 < t < \frac{T}{2}) \qquad \ddot{x} + \alpha^2 x = \mu g \quad x_{ih}(t) = x_h(t) + x_p(t) \qquad x_h(t) = C_1\cos\alpha t + C_2\sin\alpha t$

$x(t) = (x_0 - f_0)\cos\alpha t + f_0 \qquad x_p(t) = f_0$

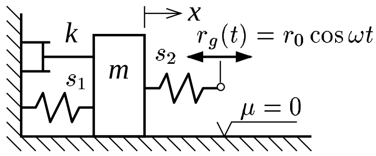
$\rightarrow \quad \dot{x} > 0 \quad (\frac{T}{2} < t < T) \qquad \ddot{x} + \alpha^2 x = -\mu g \qquad x(\frac{T}{2}) = -x_0 + 2f_0 \quad \dot{x}(\frac{T}{2}) = 0$

$x(t) = (x_0 - 3f_0)\cos\alpha t - f_0$

Gerjesztett rezgések (harmonikus gerjesztéssel)



$$m\ddot{x} + k\dot{x} + sx = F_0 \cos\omega t$$



$$m\ddot{x} + k\dot{x} + (s_1 + s_2)x = s_2 r_0 \cos\omega t$$

$$\boxed{\ddot{x} + 2D\alpha\dot{x} + \alpha^2 x = f_0\alpha^2\cos\omega t} \qquad x_h(t) = e^{-D\alpha t}(C_1\cos\gamma t + C_2\sin\gamma t) = Be^{-D\alpha t}\sin(\gamma t + \delta)$$

$$x_{ih}(t) = x_h(t) + x_p(t) \qquad \underline{x_p(t) = K\cos\omega t + L\sin\omega t = X\cos(\omega t - \vartheta)}$$

$$K = f_0\frac{1-\lambda^2}{(1-\lambda^2)^2 + 4D^2\lambda^2} \qquad N = \frac{\sqrt{K^2+L^2}}{|f_0|} = \frac{1}{\sqrt{(1-\lambda^2)^2 + 4D^2\lambda^2}} \qquad X = Nf_0$$

$$L = f_0\frac{2D\lambda}{(1-\lambda^2)^2 + 4D^2\lambda^2} \qquad \operatorname{tg}\vartheta = \frac{L}{K} = \frac{2D\lambda}{1-\lambda^2} \qquad \lambda = \frac{\omega}{\alpha}$$

szinuszos gerjesztés esetén: $\boxed{\ddot{x} + 2D\alpha\dot{x} + \alpha^2 x = f_0\alpha^2\sin\omega t}$ $\quad \underline{x_p(t) = K^\star\cos\omega t + L^\star\sin\omega t = X\sin(\omega t - \vartheta)}$

$(K^\star,\, L^\star) = (-L,\, K) \quad \operatorname{tg}\vartheta = -\frac{K^\star}{L^\star} = \frac{2D\lambda}{1-\lambda^2}$

az $N(\lambda)$ függvény csúcsponti koordinátái: $(\lambda^\star,\, N^\star) = \left(\sqrt{1-2D^2},\, \frac{1}{2D\sqrt{1-D^2}}\right)$

Másodfajú Lagrange–egyenlet... (megtanulandó!)

Többszabadságfokú lengőrendszerek mátrix differenciál egyenlete: $\boxed{\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\dot{\mathbf{q}} + \mathbf{S}\mathbf{q} = \mathbf{Q}(t)}$

Linearizált rendszer: a statikus egyensúlyi helyzet körüli kis lengéseket vizsgáljuk (q_i).

$$\mathbf{q} \equiv [q_i], \quad \mathbf{M} \equiv [m_{ij}] = \left[\frac{\partial^2\mathcal{T}}{\partial\dot{q}_i\partial\dot{q}_j}\right], \quad \mathbf{S} \equiv [s_{ij}] = \left[\frac{\partial^2\mathcal{U}}{\partial q_i\partial q_j}\right], \quad \mathbf{K} \equiv [k_{ij}] = \left[\frac{\partial^2\mathcal{D}}{\partial\dot{q}_i\partial\dot{q}_j}\right], \quad \mathbf{Q} = \left[\frac{\partial\delta\mathcal{P}}{\partial\delta\dot{q}_i}\right] = \left[\sum\nolimits_k \mathbf{F}_k\frac{\partial\mathbf{r}_k}{\partial q_i}\right]$$

Csillapítatlan szabad lengés ($k_{ij} = 0,\ \mathbf{Q}(t) = \mathbf{0}$)

$\boxed{\mathbf{M}\ddot{\mathbf{q}} + \mathbf{S}\mathbf{q} = \mathbf{0}}$ vagy $\mathbf{C}\mathbf{M}\ddot{\mathbf{q}} + \mathbf{I} = \mathbf{0}$

$\underline{\mathbf{q}_h(t) = \mathbf{A}e^{i\alpha t}}$ -t behelyettesítve:

$(\mathbf{S} - \alpha^2\mathbf{M})\,\mathbf{A} = \mathbf{0}$ vagy $(\mathbf{I} - \alpha^2\mathbf{C}\mathbf{M})\,\mathbf{A} = \mathbf{0}$

frekvencia egyenlet: $\boxed{\det(\mathbf{S} - \alpha^2\mathbf{M}) = 0}$ vagy $\det(\mathbf{I} - \alpha^2\mathbf{C}\mathbf{M}) = 0$

lengésképek: $(\mathbf{S} - \alpha_i^2\mathbf{M})\mathbf{A}_i = \mathbf{0}$ vagy $(\mathbf{I} - \alpha_i^2\mathbf{C}\mathbf{M})\mathbf{A}_i = \mathbf{0}$

modális mátrix: $\mathbf{T} = [c_1\mathbf{A}_1\, c_2\mathbf{A}_2 \ldots c_n\mathbf{A}_n] \equiv [\mathbf{A}_i]\mathbf{c} \quad c_i = \frac{1}{\sqrt{\mathbf{A}_i^\top\mathbf{M}\mathbf{A}_i}}$

Csillapítatlan harmonikusan gerjesztett lengés ($k_{ij} = 0,\ \mathbf{Q}(t) = \mathbf{F}_0\cos\omega t$)

DE: $\boxed{\mathbf{M}\ddot{\mathbf{q}} + \mathbf{S}\mathbf{q} = \mathbf{F}_0\cos\omega t}$

KF: $\mathbf{q}(0) = \mathbf{q}_0,\ \dot{\mathbf{q}}(0) = \boldsymbol{\beta}_0$

$\mathbf{q}(t) = \mathbf{q}_h(t) + \mathbf{q}_p(t)$

$\underline{\mathbf{q}_p(t) = \mathbf{L}\cos\omega t}$: $(\mathbf{S} - \omega^2\mathbf{M})\,\mathbf{L}\cos\omega t = \mathbf{F}_0\cos\omega t \Rightarrow \boxed{\mathbf{L} = (\mathbf{S} - \omega^2\mathbf{M})^{-1}\,\mathbf{F}_0}$

$\mathbf{q}(t) = \sum_{i=1}^n C_i\mathbf{A}_i\sin(\alpha_i t + \delta_i) + \mathbf{L}\cos\omega t \quad (\text{KF} \Rightarrow C_i,\ \delta_i)$

Csillapított harmonikusan gerjesztett lengés: $\boxed{\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\dot{\mathbf{q}} + \mathbf{S}\mathbf{q} = \mathbf{F}_0\cos\omega t}$

<u>Arányos csillapítás</u> esetén: $\mathbf{K} = k_M\mathbf{M} + k_S\mathbf{S}$, Modális transzformáció: $\underline{\mathbf{q} = \mathbf{T}\mathbf{y}} \Longrightarrow$

$$\mathbf{T}^\top\mathbf{M}\mathbf{T}\ddot{\mathbf{y}} + \mathbf{T}^\top\mathbf{K}\mathbf{T}\dot{\mathbf{y}} + \mathbf{T}^\top\mathbf{S}\mathbf{T}\mathbf{y} = \mathbf{T}^\top\mathbf{Q}(t) \equiv [Y_i(t)] \Longrightarrow \underline{\ddot{y}_i + 2D_i\alpha_i\dot{y}_i + \alpha^2 y_i = Y_i(t)} \quad D_i = \tfrac{1}{2}(k_M/\alpha_i + k_S\alpha_i)$$

<u>Nem arányos csillapítás</u> esetén:

$$\mathbf{q}_p(t) = \mathbf{L}\cos\omega t + \mathbf{N}\sin\omega t \Longrightarrow \begin{bmatrix} -\omega^2\mathbf{M} + \mathbf{S} & \omega\mathbf{K} \\ -\omega\mathbf{K} & -\omega^2\mathbf{M} + \mathbf{S} \end{bmatrix}\begin{bmatrix} \mathbf{L} \\ \mathbf{N} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_0 \\ \mathbf{0} \end{bmatrix} \Longrightarrow \mathbf{L},\ \mathbf{N}$$